An Efficient Dynamic Response Optimization Using the Design Sensitivities Approximated Within the Estimate Confidence Radius

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In order to reduce the expensive CPU time for design sensitivity analysis in dynamic response optimization, this study introduces the design sensitivities approximated within estimated confidence radius in dynamic response optimization with ALM method. The confidence radius is estimated by the linear approximation with Hessian of quasi-Newton formula and qualifies the approximate gradient to be validly used during optimization process. In this study, if the design changes between consecutive iterations are within the estimated confidence radius, then the approximate gradients are accepted. Otherwise, the exact gradients are used such as analytical or finite differenced gradients. This hybrid design sensitivity analysis method is embedded in an in-house ALM based dynamic response optimizer, which solves three typical dynamic response optimization problems and one practical design problem for a tracked vehicle suspension system. The optimization results are compared with those of the conventional method that uses only exact gradients throughout optimization process. These comparisons show that the hybrid method is more efficient than the conventional method. Especially, in the tracked vehicle suspension system design, the proposed method yields 14 percent reduction of the total CPU time and the number of analyses than the conventional method, while giving similar optimum values.

Key Words : Dynamic Response Optimization, Approximate Design Sensitivities

1. Introduction

Recently, there has been a growing interest in using approximate information during numerical optimization process in order to reduce the number of expensive analyses. Among of them, Lu (1992) modified Broyden's secant formula to enhance the accuracy of the approximate gradients and introduced them in trajectory optimization. Also, Kodiyalam (1997) used Lu's approximate gradient in sequential approximate optimization for the aerospace structure design. However, they did not propose a general guideline to validly use the approximate gradient during the optimization process. Lu only empirically recommended that the approximate gradients could be used after a number of iterations with exact gradients.

In order to reduce the total CPU time for design sensitivity analysis in dynamic response optimization, this study presents an estimated confidence radius to validate the approximate gradients and suggests a general numerical procedure to automatically switch using approximate gradients and using exact gradients during

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optimization process.

Dynamic response optimization problems have time dependent constraints that should be satisfied over the entire time interval. This implicit nature makes their treatment and gradient evaluation expensive. Due to this, optimization methods and design sensitivity analysis methods that have proven to be efficient for other problems tend to be expensive and inefficient for dynamic response problems (Paeng and Arora, 1989; Chahanda and Arora, 1994).

In Sec. 2, dynamic response optimization is reviewed from the views of optimization methods and design sensitivity analysis methods, respectively. This enables one to understand why this study is important in dynamic response optimization. Section 3 reviews Lu's modified method for approximating gradients and explains to estimate the confidence radius for the approximate gradient and using it during optimization process. Section 4 describes, in the context of augmented Lagrange multiplier (ALM) method with a quasi-Newton sub-optimizer, the numerical procedure of dynamic response optimization using the approximate design sensitivities approximated within the estimated confidence radius clarified in Sec. 3. Section 5 shows the numerical performances of the proposed method by solving three typical dynamic response optimization problems and one practical design problem for a tracked vehicle suspension system and comparing the optimization results with those of the conventional method. Section 6 presents the conclusions of this study.

2. Dynamic Response Optimization with ALM Method

2.1 Review of dynamic response optimization

This section reviews the early studies for dynamic response optimization. First, the optimization methods are examined. Then, the design sensitivity analysis methods are secondly observed.

From view of optimization methods, although some effective procedures have been proposed and evaluated for treating these dynamic response constraints with primal optimization methods (Hsieh and Arora, 1985; Lim and Arora, 1987) for large-scale applications, these methods still involve substantial computational effort. Thus, augmented Lagrange multiplier methods are widely used in dynamic response optimization (Paeng and Arora, 1989; Chahanda and Arora, 1994; Kim and Choi, 1998) because it has following two merits in dynamic response optimization. First, the augmented Lagrangian does not need any artificial treatments for time dependent constraints, because the augmented Lagrangian can be constructed by summing up all the constraints and integrating them over the given time interval. Second, the augmented Lagrange multiplier method, unlike primal optimization methods, does not assume the Lagrange multiplier for each time dependent constraint to be constant over the time interval. It can fundamentally generate Lagrange multipliers for the time dependent constraint over the time interval.

From view of design sensitivity analyses, in order to find an efficient method for evaluating the gradient vector of the augmented Lagrangian, Chahande and Arora (1994) have compared three design sensitivity analysis methods such as adjoint variable (AVM) method, direct differentiation method (DDM) and finite difference method (FDM). They observed that about 98 percent of the total CPU time was used for design sensitivity analysis and dynamic system analysis in dynamic response optimization, and concluded that DDM was n times more expensive than AVM and FDM might be competitive if the differential equation can be solved efficiently. The reason for these was that the differential equation system for the adjoint variable became stiff. Thus, very small steps were taken in the variable step differential equation solver. Also, the interpolation of several quantities was required for using step-by-step integration in the adjoint equations. They concluded that these resulted in a tremendous amount of computation for solving the adjoint equations. However, FDM needed less additional computational efforts than two analytical methods (DDM, AVM). This could reduce the total CPU time and enables FDM to be competitive in dynamic response optimization.

Consequently, for large scaled dynamic response optimization, it seems that ALM algorithm combined with FDM may be more efficient and effective than other methods. Also, in order to improve the efficiency of dynamic response optimization, it is a key to reduce the CPU time for design sensitivity analysis. Thus, we introduce the approximate design sensitivity information for efficient dynamic response optimization.

2.2 ALM method for dynamic response optimization

To present ideas of ALM method for dynamic response optimization, a simple dynamic response optimization problem is considered as follows:

$$\text{minimize } \psi_0(b)=\max_{t\in[0,T]} f_0(b,\ z,\ t) \tag{1}$$

$$\text{subject to } \psi_i(b,\ z,\ t)\le 0,\ 0\le t\le T,\ i=1,\cdots,m \tag{2}$$

$$b^L\le b\le b^U \tag{3}$$

where $b\in R^n$ is a vector of design variables, $z\in R^k$ is a vector of generalized velocities and displacements, t and T denote time and the final time, respectively, f_0 is the dynamic response of interest, m is the number of time dependent constraint functions, and b^L and b^U are the vectors of lower and upper values on design variables.

The ALM method sequentially minimizes the augmented Lagrangian of Eq. (4) which is convexfied by adding a penalty term to the Lagrangian (Kim and Choi, 1998; Kim and Choi 2000).

$$A(b,\ z,\ \mu,\ r)=\psi_0(b)+\int_0^T\sum_{i=1}^m[\mu_i(t)\Omega_i(b,\ z,\ t) + \frac{1}{2}r_i\Omega_i(b,\ z,\ t)^2]\,dt \tag{4}$$

where $\Omega_i(b,\ z,\ t)=\max[\psi_i(b,\ z,\ t),\ -\mu_i(t)/r_i]$; $\mu_i(t)>0$ and $r_i>0$ are the Lagrange multiplier function and the penalty parameter for the i^{th} time dependent constraint, respectively. This augmented Lagrangian is sequentially minimized as the pseudo-objective function of an unconstrained minimization sub-problem. Hence, the performance of ALM method is fully dependent on those of the adapted unconstrained optimization algorithm and it's line search algorithms.

Recently, Kim and Choi (1998) suggested an efficient algorithm to improve efficiency of a dynamic response optimization with ALM method. They used the approximated augmented Lagrangian for efficient line search and exact augmented Lagrangian for accurate determining search direction. In the k^{th} iteration of unconstrained optimization phase, they linearly approximated cost and constraint functions separately, projected them on the search direction vector S_k and composed of them as Eq. (5).

$$\tilde{A}(\alpha)=\tilde{\psi}_0(\alpha) + \int_0^T\left[\sum_{i=1}^m\left\{\mu_i(t)\tilde{\Omega}_i(\alpha,\ t)+\frac{1}{2}r_i\tilde{\Omega}_i(\alpha,\ t)^2\right\}\right]dt \tag{5}$$

where

$$\tilde{\psi}_0(\alpha)=\psi_0(b_k)+\{\nabla\psi_0(b_k)\cdot s_k\}\alpha \tag{6a}$$

$$\tilde{\Omega}_i(\alpha,\ t)=\max\{\tilde{\psi}_i(\alpha,\ t),\ -\mu_i(t)/r_T\} \tag{6b}$$

$$\tilde{\psi}_i(\alpha,\ t)=\psi_i(b_k,\ z_k,\ t)+\{\nabla\psi_i(b_k,\ z_k,\ t)\cdot S_k\}\alpha \tag{6c}$$

and $\sim$ means approximate ones. They theoretically showed that this approximated augmented Lagrangian had almost second-order accuracy near the optimum although cost and constraint functions were linearly approximated. Also, they numerically examined that their algorithm was more efficient than former ALM methods (Paeng and Arora, 1989; Chahanda and Arora, 1994) for dynamic response optimization. One may refer to Kim and Choi (1998) for detailed information on their approach.

This study extends their method to use the approximated gradient with trust radius during optimization process. Section 3 describes the basic concept for the approximated gradients and to estimate trust radius to validate the approximated gradients during optimization process.

3. The Confidence Radius to Validate the Approximate Design Sensitivities During Optimization Process

3.1 Review of gradient approximation methods for optimization

Broyden (Dennis and Schnabel, 1996) proposed a secant method to approximate Jacobian matrix for solving systems of nonlinear equations ($F(x)=0$) as

$$J_{k+1}=J_k+\frac{(y_k-J_k\Delta x_k)\Delta x_k^T}{\Delta x_k^T\Delta x_k} \tag{7}$$

where J_k represents the approximate Jacobian at x_k, $y_k=F(x_{k+1})-F(x_k)$, and $\Delta x_k=x_{k+1}-x_k$.

Recently, Lu expanded Eq. (7) to approximate the gradients in nonlinear programming for the optimal control problem (Lu Ping, 1992). He observed in his applications that changes in design variables became small after a number of iterations. Thus, he used the approximate gradient instead of FDM after the first few number of iterations.

In order to approximate the gradient at the new design x_{k+1}, he simplified Eq. (7) as

$$g_{k+1}=g_k+\frac{(\Delta F-g_k^T\Delta x_k)}{\Delta x_k^T\Delta x_k}\Delta x_k \tag{8}$$

where $\Delta F=F(x_{k+1})-F(x_k)$, and then expanded $F(x)$ in the neighborhood of x_{k+1} by second-order approximation and modified Eq. (8) as

$$g_{k+1}=g_k+\frac{(\Delta F+0.5\Delta x_k^T G_{k+1}\Delta x_k-g_k^T\Delta x_k)}{\Delta x_k^T\Delta x_k}\Delta x_k \tag{9}$$

where G_{k+1} is the Hessian at x_{k+1}. Also, in order to avoid computing G_{k+1} itself in Eq. (9), he introduced the first-order approximation as

$$g_{k+1}-g_k=G_{k+1}\Delta x_k \tag{10}$$

for small Δx_k. Multiplying Eq. (10) by Δx_k^T and approximating $g_{k+1}-g_k$ by Eq. (9) leaded to

$$0.5\Delta x_k^T G_{k+1}\Delta x_k=\Delta F-g_k^T\Delta x_k \tag{11}$$

Substituting Eq. (11) into Eq. (9) gave rise to

$$g_{k+1}=g_k+2\frac{(\Delta F-g_k^T\Delta x)}{\Delta x_k^T\Delta x_k}\Delta x_k \tag{12}$$

In this approach, Lu theoretically showed that Eq. (12) was more accurate than Broyden's original formula if $\|\Delta x_k\|$ is small, although two equations only differ in the coefficient. For more detailed information of Lu's approximation, one may refer to Lu (1992).

However, in order to validate using Eq. (12) during optimization process, Lu did not recommend how less design change was valid for the small Δx_k in Eq. (10) and how much iteration were a few iterations. He just empirically recommended that Eq. (12) should be followed by first employing FDM for a few iterations, since Δx and ΔF underwent relatively large changes at the beginning of optimization process. We think that this recommendation is not enough to certify using Eq. (12) during optimization process. Thus, we will suggest a guideline to check the validity of using Eq. (12) during optimization process in section 3.2.

3.2 Estimating the confidence radius for validating approximate gradients

In order to check the validity for using Eq. (12) during optimization process, we propose a guideline, which employs an approximated Hessian for determining how less design change was valid for the small Δx_k in Eq. (10). The approximated Hessian can be easily evaluated by employing one of quasi-Newton formulae. We believe that it is not severe restriction since quasi-Newton algorithms can directly solve unconstrained optimization problems and sequentially solve constrained optimization problems as sub-optimizer of ALM and SUMT methods.

Now, we modify the first-order approximation of Eq. (10) as

$$g_{k+1}-g_k=B_{k+1}\Delta x_k \tag{13}$$

for small Δx_k. B_{k+1} is an approximated Hessian updated by a quasi-Newton formula. Eq. (13) is sometimes called the quasi-Newton condition. If we know g_k, g_{k+1} and B_{k+1}, we may evaluate Δx_k to validate Eq. (13) as

$$\Delta x_k=B_{k+1}^{-1}(g_{k+1}-g_k) \tag{14}$$

However, we don't use Eq. (14) directly to evaluate Δx_k, because B_{k+1} is sequentially approximated by using B_k, $\gamma=g_{k+1}-g_k$ and $\Delta x_k=x_{k+1}-x_k$. Hence, we use B_k in place of B_{k+1} in Eq. (14) since they have the similar meaning from the viewpoint of Taylor's series expansion. Also, we employ the inverse Hessian approximation formula $(H_k=B_k^{-1})$ to avoid solving linear equation of (14). This relation is shown using the Sherman-Morrison-Woodbury formula (pp. 198 ~203 from Dennis and Schnabel, 1996).

Now, we can estimate the confidence radius of (15) for the approximated gradient g_{k+1} to satisfy the first-order approximation of Eq. (10).

$$\rho_k = \|H_k(g_{k+1} - g_k)\|_\infty \qquad (15)$$

Consequently, we check the following inequality condition of (16) before we adapt the approximated gradient g_{k+1} during optimization process.

$$\|\Delta x_k\|_\infty \leq \rho_k \qquad (16)$$

If this inequality condition is satisfied, we regard that the k^{th} design change is small enough to satisfy the first-order approximation of Eq. (10). Otherwise, exact gradient is employed.

The detailed numerical procedure of a quasi-Newton algorithm with approximated gradient is fully described in Sec. 4.

4. Computational Procedure

4.1 ALM method for dynamic response optimization

In this paper, we use the approximate gradient vectors in solving the constrained dynamic response optimization problems. Thus, we first explain following ALM method for dynamic response optimization:

Step 1. Select an initial design variable vector b^0, an initial Lagrange multiplier vector μ^0, and an initial penalty parameter vector r^0. Set $q=0$.

Step 2. Starting from b^q, minimize $A(b, z, \mu^q, r^q)$ of Eq. (4) subject to $b^L \leq b \leq b^U$, where b^L and b^U are the vectors of lower and upper limit values on design variables, respectively (Kim and Choi, 1998). Let the solution be b^{q+1}.

Step 3. At the optimum b^{q+1}, if the peak of every time dependent constraint is lower than a specified tolerance ε_1 and the relative reduction of the cost value is less than a specified tolerance ε_2, then stop. Otherwise, go to Step 4.

Step 4. Update the Lagrange multipliers by $\mu_i^{q+1}(t) = \mu_i^q(t) + r_i^q \max[\psi_i(b^{q+1}, z^{q+1}, t), -\mu_i^q(t)/r_i^q]$, $i=1,\cdots,m$,

and the penalty parameters based on the Lagrange multiplier values and the degrees of satisfaction of the constraint functions (Kim and Choi, 2000). Go to Step 2 with $q=q+1$.

Constraints are normalized in this study for one or more of the constraints not to dominate the augmented Lagrangian. Also, due to similar reason, design variables are normalized at the beginning of each unconstrained minimization. Initial Lagrange multipliers are assumed to be zero. Based on our numerical experience, initial penalty parameters are set to be 1, 100, and 10 for lightly violated ($0 < \psi_i \leq 1$), heavily violated ($\psi_i \geq 1$), and feasible ($\psi_i \leq 0$) constraints, respectively. We assigned a relatively large penalty parameter value of 10 for initially feasible constraints since they tend to be severely violated after the first ALM iteration due to their lack of contribution to the penalty term.

We elaborate on our approach to the unconstrained minimization step (Step 2 of the above algorithm) in the next section.

4.2 Unconstrained minimization with approximate gradients

The performance of ALM method is fully dependent on those of the unconstrained minimization algorithm and it's line search algorithm adapted for Step 2 of the above ALM algorithm. Thus, we expand the efficient unconstrained algorithm of Kim and Choi (1998) in order to use the approximated design sensitivities of Eq. (12) during optimization process. As described in section 3.1, Lu used the approximated gradients followed by FDM after a number of iterations. However, our suggested algorithm automatically switches the approximated gradient and exact gradient for some conditions. The expanded algorithm is as follows:

Step 1. Start with b_0. (b_0 is the same as b^q in the ALM algorithm.) Solve for dynamic response and calculate the function and gradient values of $A(b_0)$. Let these values be denoted by A_0 and ∇A_0, respectively. Set $k=0$, $\eta=0$, $\theta_1=0.05$, $\theta_2=0.4$,

$Ig=0$. Also, $\gamma=\max\{1, ndv/3\}$ for $ndv>5$ or $\gamma=\max\{1, ndv/2\}$ for $ndv\leq 5$ is set.

Step 2. Determine the search direction S_k by employing one of quasi-Newton methods. (This study uses BFGS formula for updating inverse Hessian.)

Step 3. Construct the approximate augmented Lagrangian $\tilde{A}(\alpha)$ of Eq. (5) and perform line search to minimize $\tilde{A}(\alpha)$ subject to $\alpha\leq\alpha^U$. Let the solution be α^*. For brevity, the detailed line search algorithm of Kim and Choi (1998) is not described in this paper. For more detailed procedure of their algorithm, one should be refer to Kim and Choi (1998).

Step 4. If $\alpha^*=0$ and $\eta=1$, then stop since the failure of a steepest descent move indicates that the minimum has been found. However, if $\alpha^*=0$ and $\eta=0$, then go to Step 5. Otherwise, update design variable as $b_{k+1}=b_k+\alpha_k{}^*S_k$, and go to Step 6.

Step 5. If $Ig=0$, then restart the process by resetting $S_k=-\nabla A_k$ and go to Step 3 with $\eta=1$. Otherwise, evaluate the exact gradient vector of $\nabla A(b_k)$ and let $\nabla A_k=\nabla A(b_k)$. Then, restart the process by resetting $S_k=-\nabla A_k$ and go to Step 3 with $\eta=1$ and $Ig=0$.

Step 6. Solve for dynamic response and calculate function value $A(b_{k+1})$ and evaluate $\|\Delta b_k\|_\infty$. Also, approximate $\nabla\tilde{A}(b_{k+1})$ by Eq. (12), estimate ρ_k by Eq. (15), and check the inequality condition of (16). If $\{\|\Delta b_k\|_\infty\leq\rho_k$ and $k\geq\gamma\}$ or $\{\|\Delta b_k\|_\infty\leq\theta_1\|b_k\|_\infty$ and $k<\gamma\}$, then accept approximated gradients, let $\nabla A_{k+1}=\nabla\tilde{A}(b_{k+1})$, and go to Step 7 with $Ig=1$. Otherwise, evaluate exact gradient vector $\nabla A(b_{k+1})$, let $\nabla A_{k+1}=\nabla A(b_{k+1})$, and go to Step 7 with $Ig=0$.

Step 7. If$[\{|A_{k+1}-A_k|\leq\varepsilon_4$ or $|A_{k+1}-A_k|/|A_{k+1}|\leq\varepsilon_5$ for two successive iterations$\}$ and $\{\|\nabla A_{k+1}\|\leq 0.2\|\nabla A_0\|\}]$, then stop. Also, if $\|\nabla A_{k+1}\|\leq\varepsilon_2$ and $Ig=0$, then stop. However, if $\|\nabla A_{k+1}\|\leq\varepsilon_2$ and $Ig=1$, then evaluate the exact gradient vector of $\nabla A(b_{k+1})$ and recheck whether $\|\nabla A(b_{k+1})\|\leq\varepsilon_2$ or not. If this condition is satisfied, then stop. Otherwise, let $\nabla A_{k+1}=\nabla A(b_{k+1})$ and go to Step 8 with $Ig=0$.

Step 8. Update the inverse Hessian using BFGS formula and go to Step 2 with $\eta=0$.

In the above computational procedure, the symbol $\eta=1$ denotes that the search direction S_k is reset to the steepest descent direction because S_{k-1} is failed to improve design. Also, the symbol $Ig=1$ denotes that the current gradient vector ∇A_k is constructed by the approximate design sensitivities. In the step 7, the value of 0.2 is determined from numerical experience.

If it is well defined, and if $\Delta x^1, \Delta x^2, \cdots, \Delta x^k$ are independent, then the Broyden family of quasi-Newton methods with exact line searches terminates after $k\ll ndv$ iterations on a quadratic function. Also, if $k=ndv$, then $H_{k+1}=G^{-1}$ (pp. 203-207 from Dennis and Schnabel, 1996). Although these characteristics encourage using Eq. (15), the estimated confidence radius of Eq. (15) may be unstable for $k\ll ndv$ iterations, because one may try to use the above algorithm with inexact line search in solving non-quadratic problems.

In order to overcome this situation, safeguard is included in Step 6 of the above computational procedure. This safeguard represents that, when k is less than γ, the approximate gradient is adapted only for $\|\Delta b_k\|_\infty\leq\theta_1\|b_k\|_\infty$ because the approximate Hessian is not fully developed.

The values of γ and θ_1 are empirically defined in Step 1 of the above computational procedure. Although the algorithm is insensitive to their change, it is recommended that the value of γ should be less than 10 for effective using approximate gradients. Also, the value of θ_1 should be set within $0.01-0.05$ because FDM usually uses 1 percent perturbation for each design variable in engineering optimization problems. This represents the $1\sim 5$ percent design changes are regarded as a small design changes in this study.

5. Numerical Studies

In order to show numerical performance of the proposed algorithm, a dynamic response optimization program having two options for employing gradient vectors is developed. This program is based on the computational procedures described in Sec. 4 and Kim and

Table 1 Optimization results for non-linear impact absorber with $\omega=2$

	Initial	Hybrid Method		Conventional Method*	
	Design	With FDM	Will DDM	With FDM	With DDM
b_1	0.5000	0.5971	0.5971	0.5971	0.5971
b_2	0.5000	0.5972	0.5972	0.5972	0.5972
Objective Value	0.5945	0.5972	0.5972	0.5972	0.5972
NG	-	8	9	15	15
NF	-	17	17	15	15
NA	-	8	8	-	-
NT(=NG*NDV+NF)	-	33	-	45	-

* ALM method with approximated augmented Lagrangian suggested by Kim and Choi(1998)

Choi (1998, 2000 and 2001). Then it is applied to solve three typical dynamic response optimization problems and one practical dynamic response optimization problem.

Three typical problems are a single degree-of-freedom nonlinear impact absorber design, a two degree-of-freedom linear vibration isolator design, and a five degree-of-freedom vehicle suspension system design, which are fully described in Haug and Arora (1979).

One practical dynamic response optimization problem is the design of Hydro-pneumatic Suspension Units (HSU) that minimize the maximum acceleration at a mass center of a tracked vehicle that run over a 36 cm (14 inch) bump with 40 km/h, while satisfying design limits for wheel travels, the maximum acceleration at a mass center, track tension, static balance of six wheels.

In this study, for the small-scaled three typical problems, the Runge-Kutta fifth-and sixth-order method is used for dynamic analysis and DDM for design sensitivity analysis. However, for a complicated tracked vehicle model, RecurDyn 1.0 based on a recursive BDF method (Han, Bae and Yoo, 1999; Bae, Kim, Yoo and Suh, 1999) and FDM are employed for dynamic analysis and design sensitivity analysis. Simpson's rule is used to carry out the numerical integration in augmented Lagrangian of Eq. s (4) and (5). Through this study, the following values are specified for the convergence tolerances: $\varepsilon_1=1\times10^{-4}$, $\varepsilon_2=1\times10^{-1}$ and $\varepsilon_3=\varepsilon_4=\varepsilon_5=1\times10^{-2}$.

In Tables 1~4 that lists the optimization results for four design problems, Hybrid Method

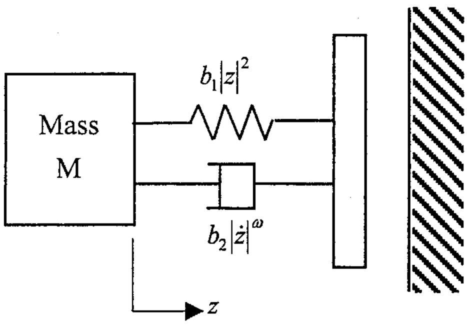


Fig. 1 Nonlinear impact absorber

is the proposed method that automatically switch using approximate gradients and using exact gradients during optimization process, and Conventional Method is the conventional method that uses only exact gradients throughout optimization process. NF and NG denote the number of function and gradient evaluations. NA denotes the number of approximate gradient being adapted in Hybrid Method. Also, NT symbolizes the total number of function evaluations including that evaluated for FDM only when FDM is employed for design sensitivity analysis.

5.1 Three typical dynamic response optimization problems

5.1.1 Single degree-of-freedom nonlinear impact absorber

A single degree-of-freedom nonlinear impact absorber shown in Fig. 1 has a fixed mass and nonlinear spring and damper elements. The spring and damper coefficients b_1 and b_2 are taken as design variables. The system impacts a fixed

Table 2 Optimization results for dynamic absorber

	Initial	Hybrid Method		Conventional Method*	
	Design	With FDM	Will DDM	With FDM	With DDM
b_1	1.600	1.3552	1.3521	1.3484	1.3491
b_2	0.020	0.0199	0.0201	0.0215	0.0209
Objective Value	3.189	2.3721	2.3691	2.3684	2.3680
NG	-	4	4	8	8
NF	-	14	12	14	13
NA	-	6	4	-	-
NT(=NG*NDV+NF)	-	22	-	30	-

* ALM method with approximated augmented Lagrangian suggested by Kim and Choi(1998)

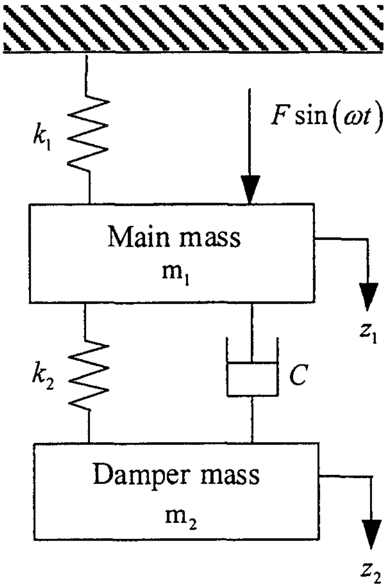


Fig. 2 Vibration absorber

barrier at time $t=0$ with a given initial velocity. The objective is to find b_1 and b_2 that minimize the maximum acceleration of the mass subject to a constraint on displacement of the mass. This example problem is solved for $\omega=2$.

Optimization results for four comparison cases are listed in Table 1. The optimization results show that Hybrid Method is more efficient than Conventional Method regardless of DSA method. In case of employing FDM, Hybrid Method gives 26 percent reduction of function evaluations than Conventional Method. Also, when DDM is used, the number of design sensitivity analyses for Hybrid Method is found to be 60 percent of that of Conventional Method.

5.1.2 Two degree-of-freedom linear vibration isolator

A two degree-of-freedom dynamic absorber is shown in Fig. 2. The objective is to find the damping coefficient c and spring constant k_2 that minimize the peak transient dynamic displacement of the main mass for a given excitation frequency subject to constraints on transient and steady state responses and explicit bounds on design variables.

The optimization results for the two-degree-of-freedom linear vibration isolator are given in Table 2. In this problem, although Hybrid method with FDM gives 0.1 percent inaccurate design than Conventional method with FDM, the number of design sensitivity analyses for Hybrid Method is found to be 50 percent of that for Conventional Method.

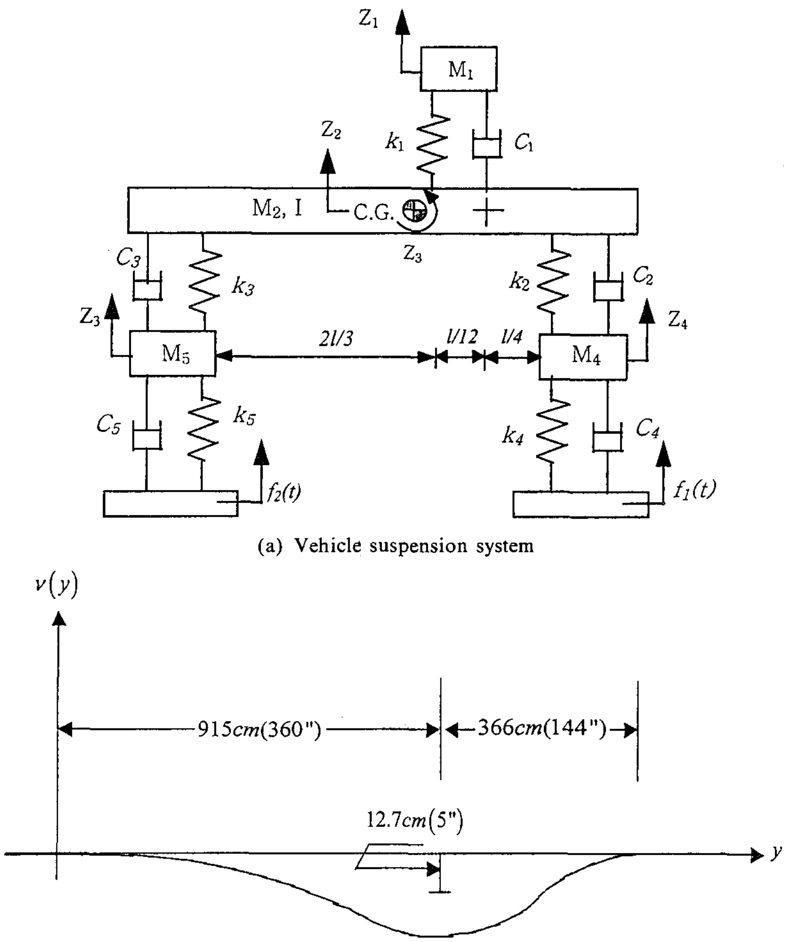
5.1.3 Five degree of freedom vehicle suspension system

Figure 3(a) shows a five degree of freedom vehicle suspension system, which is to be designed to minimize the extreme acceleration of the driver's seat for a given vehicle speed and a road surface profile shown in Fig. 3(b). This profile is a combination of two sinusoidal curves with different half-wavelengths, which represents a severe bump condition. Spring constants k_1, k_2 and k_3 and damping coefficients c_1, c_2 and c_3 of the system are chosen as design variables. The motion of the vehicle is constrained so that the relative displacements between the chassis and the driver's seat, the chassis and the front and rear axles, and

Table 3 Optimization results for vehicle suspension system running profile No. 1

	Initial	Hybrid Method		Conventional Method*	
	Design	With FDM	Will DDM	With FDM	With DDM
b_1	100.0	50.00	50.00	50.00	50.00
b_2	300.0	200.02	200.06	200.21	200.04
b_3	300.0	200.00	200.00	200.00	200.00
b_4	10.0	35.31	45.43	42.48	44.17
b_5	25.0	77.37	77.59	77.47	77.43
b_6	25.0	80.00	80.00	80.00	80.00
Objective Value	332.6	254.82	254.98	254.84	254.81
NG	-	11	12	14	16
NF	-	17	18	16	17
NA	-	6	6	-	-
NT(=NG*NDV+NF)	-	83	-	100	-
CPU Time(seconds)	-	4.28	4.65	5.19	5.83

*ALM method with approximated augmented Lagrangian suggested by Kim and Choi(1998)



(a) Vehicle suspension system

(b) Road surface profile

Fig. 3 Five-degree-of-freedom vehicle model

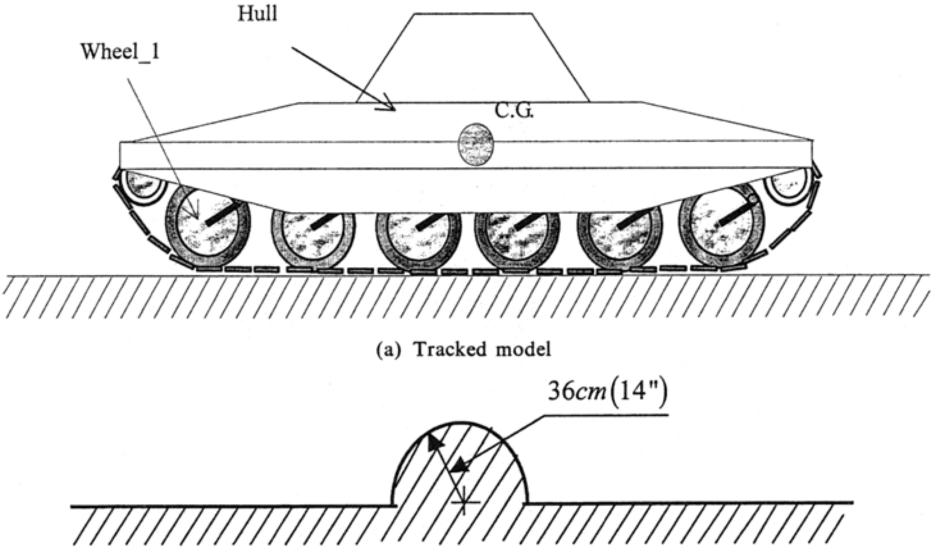


Fig. 4 Seven degree-of-freedom tracked vehicle suspension system

the road surface and the front and rear axles are within given limits. The design variables are also constrained.

Table 3 lists the optimum results for the five-degree-of-freedom vehicle system excited by profile shown in Fig. 3 (b). Hybrid Method gives 20 percent reduction of the number of design sensitivity analysis than Conventional Method. Consequently, in case of employing FDM, Hybrid Method is found to be 80 percent of that with Conventional Method.

5.2 Tracked vehicle suspension design problem

Figure 4(a) shows a tracked vehicle suspension system, which is to be designed to minimize the maximum acceleration of the mass center when the vehicle run over a bump shown in Fig. 4(b) for a given speed (40km/h). The tracked vehicle model is composed of a hull, two sprockets, six wheels with HSU suspension systems and track. Nine design variables are divided the following three groups:1) the pre-pressures for the HSU systems of 1^{st}, 2^{nd}, 5^{th} and 6^{th} wheels, 2) the track tension force in the static state, and 3) the length of a gas chamber, the pre-pressure of Bellevile springs, the inner diameter of orifice and the fluid flow at choking point for all the HSU systems.

The motion of the vehicle is constrained so that the maximum acceleration of mass center, wheel travels for the six wheels, and static reaction forces for the six wheels are within given limits. Also, the pre-pressures of HSU systems for 3^{rd} and 4^{th} wheels are within given limits. The design variables are also constrained within nearly$\pm$15 percent of the initial design.

Figure 5 shows the convergence history for the tracked vehicle suspension design problem. The abscissa represents the number of exact design sensitivity analyses, which does not include the number of approximate design sensitivities. During optimization process of both methods, wheel travel constraint for the 1^{st} wheel and constraints for balancing static reaction forces loaded for the six wheels are severely conflicted.

Table 4 lists the optimum results of both methods side by side. These show that Hybrid Method can reduce 14 percent of analyses than Conventional Method while it gives 0.2 percent greater than design than Conventional Method. Also, in the comparisons of the total CPU time between both methods for a Pentium III 550 MHz compatible computer, Hybrid method can save nearly 14 percent of CPU times. It is interesting

Table 4 Optimization results for tracked vehicle system

	Initial Design	Final Design	
		Hybrid Method	Conventional Method*
b_1	180.00	184.63	181.32
b_2	152.00	153.20	152.63
b_3	140.00	143.67	152.08
b_4	154.00	153.54	152.28
b_5	43895.00	41799.74	42728.10
b_6	0.135	0.140	0.140
b_7	115.00	121.74	135.52
b_8	0.0038	0.0041	0.0040
b_9	610.00	633.65	643.35
Objective Value	23.757	14.923	14.890
Max. Violation	0.00	0.00008	0.0001
NG	-	22	27
NF	-	34	29
NA	-	6	-
NT(=NG*NDV+NF)	-	232	272
CPU Time(seconds)	-	5549.87	6480.72

* ALM method with approximated augmented Lagrangian suggested by Kim and Choi(1998)

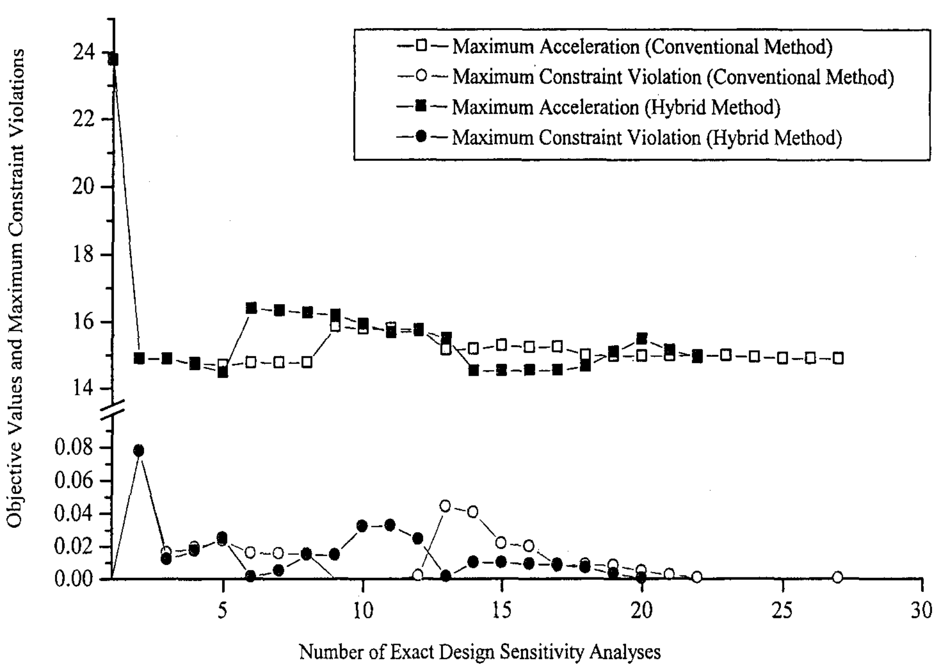


Fig. 5 Convergence history of a tracked vehicle suspension system design

that this reduction is equal to that for the comparison of number of analyses. This represents that the numerical procedure for approximating the gradient vector hardly requires computing time compared with dynamic system analysis.

Finally, Fig. 6 shows the vertical acceleration of C. G. for the tracked vehicles of the initial and final design running over the single bump shown

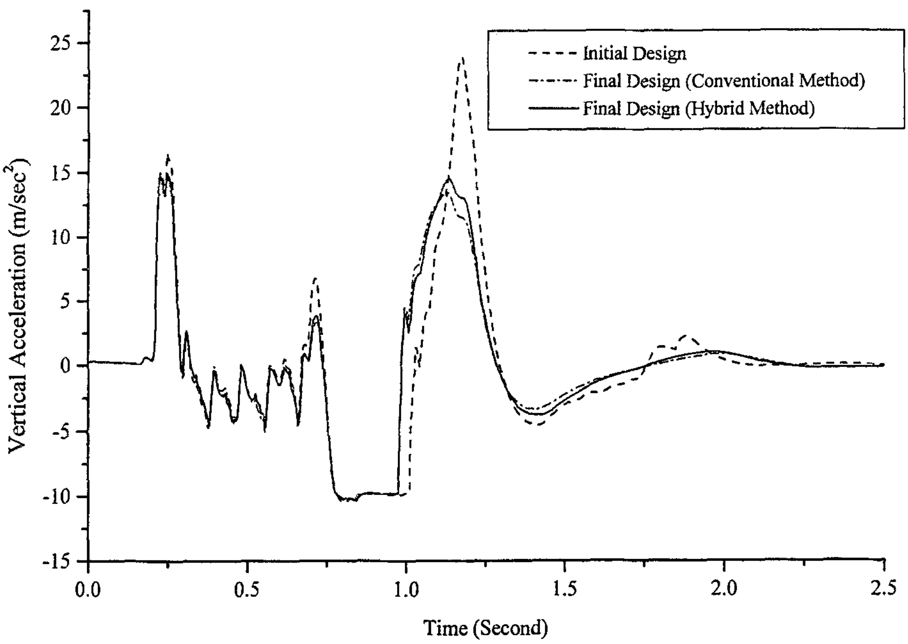


Fig. 6 Magnitudes of vertical acceleration of C.G. for vehicles of initial and final design running over the 36cm single bump

in Fig. 4 (b). Both methods show nearly equal responses.

6. Concluding Remarks

In order to validate using the approximate gradient during numerical optimization process, an effective method is presented. This study proposed the estimated confidence radius to qualify the approximate gradients before using them in the numerical optimization, and implemented it into a quasi-Newton algorithm used as a sub-optimizer of ALM method.

A computer program implementing computational procedures of the proposed Hybrid method for automatic switching approximate gradients and exact gradients is developed and applied to solve three typical dynamic response optimization problems and one practical design problem for a tracked vehicle suspension system.

For three typical design problems, the proposed method is combined with DDM and FDM, respectively. Both approaches tested are more efficient than the conventional methods that use only exact gradients throughout the optimization process. Especially, for the practical dynamic response optimization with small design range (± 15 percent of the initial design), the proposed method yields 14 percent reduction of the total CPU time and the number of analyses than the conventional method. This represents that the proposed method is effective in practical dynamic response optimization.

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